MODEL EXAMINATION 2011-12

MATHEMATICS(SCIENCE)

Maximum Score 80 Time : $2\frac{3}{4}$ hrs

HSE II (Including cool off time 15minutes)

General Instructions

- You are not allowed to write answers or discuss anything with others during cool of time.
- Cool off time is for familiarizing questions and planning answers.
- All questions are compulsory and only internal choice is allowed.
- When you select a question all sub questions must be answered from the same question itself.

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1. (i) Consider the function f:{1,2,3,4} → {10} defined by f= { (1,10),(2,10),(3,10),(4,10)}
   Is sthe function f invertible ? jusitfy with reason. (1)

   (ii) A function f:R→R defined by $f(x) = \frac{2x-3}{7}$ and $g(x) = (x+1)^2$; Find fog (x) (2)

   (iii) Let * be a binary operation defined on Q by $a*b = \frac{ab}{4}$ for a,b ∈ Q . Find the identity element of (Q ,*) (2)

2. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 5 & 4 \\ 1 & 6 \end{bmatrix}$ (i) Find the order of AB (ii) Verify that $(AB)^T = B^T A^T$ (1+2)

3. Using properties prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$ (4)

4. Consider the following system of equations x+2y+5z = 10 , x-y-z = -2 , 2x+3y-z = -11
   (i) Express this system of equations in the form Ax=B (1)
   (ii) Prove that A is non singular . (1)
   (iii) Find the values of x,y and z satisfying the above system of equations. (3)

5. Match the following (4)

A	B
$\sin^{-1}(\sin\frac{2\pi}{3})$	$\frac{4}{5}$
$\cos(\sin^{-1}\frac{3}{5})$	$\frac{\pi}{2}$
$\tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{3})$	$\frac{\pi}{3}$
$\sin^{-1}x + \cos^{-1}x$	$\frac{\pi}{4}$
	1

6. (i) Show that the function defined by $f(x) = \cos(x^2)$ is a continous function . (2)

   (ii) Find $\frac{dy}{dx}$ if $y = (\log x)^{\cos x}$ (1)

   (iii)If $y = \sin^{-1}x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ (3)

7. (i) Let $x = a(\theta + \sin\theta)$ and $y = a(1-\cos\theta)$, Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (2)

   (ii) Consider the function f(x) =x(x-2) , x ∈ [1,3] .Verify the mean value theorem for the function in [1,3] (2)

8. (i) Using differentials ,find the approximate value of $(82)^{\frac{1}{4}}$ upto 3 decimal places (3)

   (ii) Find two positive numbers x and y such that their sum is 35 and product x^2y^5 is maximum. (3)

10. Evaluate the following integrals

    (i) $\int x^2 \log x \, dx$ (ii) $\int \frac{3x-2}{(x-2)^2} dx$ (iii) $\int \frac{\cot(\log x)}{x} dx$ (2+2+2)

11. Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ (4)

12.(a) Using integration find the area of the region bounded by the triangle whose vertices are (1,0),(2,2) and (3,1) (4)

OR

(b) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}$ and the straight line $\frac{x}{2}+\frac{y}{3}=\mathbf{1}$ using integration. (4)

13. (i) Find the order and degree of the differential equation $\left(\frac{d^2s}{dt^2}\right)^2 + 3\left(\frac{ds}{dt}\right)^2 + 2\left(\frac{ds}{dt}\right)^3 + 4 = 0$ (1)

(ii) Solve the differential equation $(x^2-y^2)dx + 2xydy = 0$ (3)

14. Consider the vectors $\vec{a} = 3\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and $\vec{b} = \hat{\imath}+2\hat{\jmath}-2\hat{k}$, find

(i) $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$ (1)

(ii) $(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})$ (2)

(iii) a unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ (2)

(iv) a vector of magnitude 5 units and which perpendicular to $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ (1)

15.(a) Consider the pair of straight lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+1}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

(i) Write the direction ratios of the lines . (1)

(ii) Find the angle between the two lines . (2)

OR

(b) Findthe shortest distance between the above straight lin es. (3)

16. Find the equation of the plane passing through the intersection of the planes x+y+4z+5=0 and 2x-y+3z+6=0 and containing the point (1,0,0) (2)

17. A company manufactures two products A and B on which the profits earned per units are Rs 30/-and Rs 40/- respectively.Each product is processed on two machinesM_1 and M_2 .One unit of product A requires 1 hour of processing time on $M_1$and 2 hours on M_2 while one unit of product B requires 1 hour each on M_1 and M_2 . Machines M_1 and M_2 are available at most 10 hours and 12 hours respectively during a working day.The company wants to know how many units of each product A and B should produce to maximize the profit. To formulate a linear programming problem,

(i)Write the objective function. (ii)Write all the constraints . (1+2)

18. Solve the given linear programming problem graphically.

Minimise Z=6x+3y, Subject to: $4x+y\geq 80$, $x+y\geq 115$, $3x+y\leq 150$ $x,y\geq 0$ (3)

19. Let A and B be independent events with P(A)=0.3 and P(B) =0.4

Find (i) P(A∩B) (ii) P(AUB) (iii) P(A/B) (1+1+1)

20 (i) A bag contains 4 red and 4 black balls .Another bag contains 2red and 6 black balls. One of the bag is selected at random and a ball is drawn.If the ball drawn is red, find the probability that the ball drawn is from the first bag. (3)

(ii) There are 5% defective items in large bulk of items.Using binomial distribution find the probability that a sample of 10 items will include not more than one defective item. (2)
